Universality of Symmetry and Mixed-symmetry Collective Nuclear States

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Abstract

The global correlation in the observed variation with mass number of the $E2$ and summed $M1$ transition strengths is examined for rare earth nuclei. It is shown that a theory of correlated S and D fermion pairs with a simple pairing plus quadrupole interaction leads naturally to this universality. Thus a unified and quantitative description emerges for low-lying quadrupole and dipole strengths.

When seemingly unrelated physical quantities exhibit similar empirical behavior (a universality), one suspects a common physical genesis. One such universality was recently recognized in nuclear structure physics: The measured electromagnetic transition strength $B(E2)$ between the ground and the first 2^+ states of even–even rare earth nuclei exhibit a variation with mass number A that is similar to the variation with A of the summed orbital $B(M1)$ strengths measured in the same nuclei [1]. In this letter, we shall explore the physical implications of this universality.

The excited states associated with these $E2$ and summed $M1$ strengths (the 2_1^+ and the lowest 1^+ states, respectively) have very different collective character. The former are symmetry states, in which protons and neutrons move with phase coherence; examples are low-lying rotations and vibrations. The latter are mixed-symmetry states, in which protons and neutrons move out of phase; examples are the lowest 1^+[2] and some excited 2^+[3] states. Thus, the physical $0_1^+ \rightarrow 1_1^+$ and $0_1^+ \rightarrow 2_1^+$ transition strengths measure two distinct classes of excitation: $E2$ excitations between symmetry states and $M1$ excitations between symmetry and mixed-symmetry states (Although we address the summed B(M1) strength of 1^+ states up to 4 MeV in our calculations, the dominant transition is typically $B(M1; 0_1^+ \rightarrow 1_1^+)$). Thus, the observed correlation in these two strengths hints at a microscopic correlation for two modes which in the simplest geometrical picture appear to be rather different. Its appearance provides a new window to gain additional insights into the low energy behavior of nuclear structure.

Nuclear deformation is known to arise from the interplay of the long-range n–p quadrupole (QQ) interaction and the short-ranged pairing interaction between like nucleons, as was emphasized by Federman and Pittel [4]. However, straightforward application of the shell model for the nuclei in question is prohibitively difficult. Algebraic approaches offer approximate solutions of the shell model, and have provided some important insights into this problem. For example, the neutron–proton version of the Interacting Boson Model (IBM-2)[5] allows one to use the F-spin [6] quantum number to classify these states: symmetric with maximal F-spin and mixed-symmetry with smaller

values of F-spin. However, Rangacharyulu et. al [1] concluded that it is difficult for the IBM-2, at least under the assumption of preserving the F-spin symmetry, to account for the universality. Likewise, Ginocchio [7] suggests that with the F-spin symmetry and using the arguments from the well-known Otsuka-Arima-Iachello (OAI) mapping procedure [8] that it is not straightforward to show both the $E2$ and $M1$ saturate well before midshell (see the following figures for the data). Finally, $M1$ excitations have also been specifically and quantitatively discussed with the quasiparticle picture in the Nilsson model (e.g. [9]).

The Fermion Dynamical Symmetry Model (FDSM) [10, 11] is an $SO(8)$ or $Sp(6)$ truncation of the shell model. Since Pauli correlations generally are expected to become more important as midshell is approached, it is reasonable to ask whether or not a model such as the FDSM that incorporates the true fermionic nature of the correlated pairs can account for the similar behavior of $E2$ and $M1$ strengths. We are encouraged to pursue this question by the results from a recent paper [12] which showed that the same $Q_\pi \cdot Q_\nu$ interaction is responsible for the splitting of symmetry and the 2^+ mixed-symmetry states, and can consistently describe their transitional properties. In this letter, we shall show that such a universality between $E2$ and $M1$ strengths arises within the FDSM framework with a reasonable set of effective interaction parameters for the rare-earth nuclei. In addition, we shall demonstrate that the same calculation reproduces quantitatively the energy ratio $E(4_1^+)/E(2_1^+)$, and accounts for the more subtle deviations from universal behavior exhibited by these three quantities.

The primary building blocks of the FDSM are S and D (monopole and quadrupole) correlated fermion pairs, whose integrity in low-lying states is maintained even in the presence of the empirical single-particle energy splittings [13]. The total neutron–proton FDSM symmetry for the rare-earth nuclei is $Sp^\nu(6) \times SO^\pi(8)$. Ref. [14] demonstrates that the low-lying spectroscopy of the rare earth nuclei can be reasonably well described by a 5-parameter (pairing and QQ) FDSM Hamiltonian:

$$H = G'_{0\pi} S_\pi^\dagger S_\pi + G'_{0\nu} S_\nu^\dagger S_\nu + B'_{2\pi} P_\pi^2 \cdot P_\pi^2 + B'_{2\nu} P_\nu^2 \cdot P_\nu^2 + B_{2\pi\nu} P_\pi^2 \cdot P_\nu^2. \tag{1}$$

All the effective operators in Eq.(1) are defined in [10]. In this Hamiltonian, the FDSM quadrupole pairing interactions are also taken into account by renormalizing the parameters: $G'_{0\sigma} = G_{0\sigma} - G_{2\sigma}$ and $B'_{r\sigma} = B_{r\sigma} - G_{2\sigma}$ ($\sigma = \pi, \nu$). One generally finds that the n–p QQ interaction is significantly stronger than the pairing interaction between like nucleons.

The model space is restricted to the S–D subspace in the normal-parity shells (heritage $u = 0$, corresponding to no broken pairs). Although the particles in abnormal-parity levels are not included explicitly, they are included effectively by the constraint that there is a distribution of particles between the normal and the abnormal parity levels. The number of pairs (N_1) in the normal-parity levels is treated as a good quantum number and is calculated from a semi-empirical formula determined globally from the ground state spin of the odd-mass nuclei [10].

In the IBM-2, the F-spin algebra allows the introduction of an F-spin (Majorana) scalar interaction that has no effect on the symmetric states. Its strength is chosen to place the mixed-symmetry states at the proper energies. The energies of the low-lying symmetric 2_1^+ and 4_1^+ states are usually chosen to determine the strength of the $Q_\pi \cdot Q_\nu$ interaction. Thus in the IBM-2, the strengths of the $Q_\pi \cdot Q_\nu$ and the Majorana interactions are separately fitted to states that have different collective behaviors. On the other hand, the FDSM, and indeed most fermion models built from pairs, cannot have a closed F-spin algebra. This constraint will prevent the fermion picture to having an analogus phenomenological separation of and mixed-symmetry states. Therefore, in the fermion picture, one must describe all states, symmetric or mixed-symmetric, by a single Hamiltonian.

The five parameters in Eq. (1) are determined numerically using a gradient search within the FDU0 code [15] to best reproduce the experimental spectrum of the nuclide in question. The experimental energies used in the fit for the systematics are 2_1^+, 4_1^+, 6_1^+, 1^+, 2_2^+, 0_2^+ in Nd, Sm, Gd, Dy, and Er (some 1^+ states are not known experimentally for Sm and Nd isotopes). This procedure is carried out until a good match to the experimental

spectra and a smooth trend in particle number was found for the Hamiltonian parameters. Similar calculations have been described in [16, 10] and details of the present fitting process will be discussed in a forthcoming paper [17].

When a suitable fit is found for a particular nuclide, the correlation of the symmetry and mixed-symmetry states is given by the unified Hamiltonian of Eq. (1). We find that the QQ-interaction plays the crucial role in correlating the 2_1^+ and 1^+ states because the excitation energies of these states depend sensitively on this term. Once a suitable spectrum has been determined, the next step is to see whether there is correlation between symmetry and mixed-symmetry states caused by the same $Q_\pi \cdot Q_\nu$ strength for the electromagnetic transitions.

To compute the electromagnetic transitions, one needs the wavefunctions of the states in question and the effective transitional operators. From our unified fit, we are able to obtain these wavefunctions. The $M1$ and $E2$ effective transition operators in the FDSM are

$$T(M1)_\mu^1 = \sqrt{\frac{3}{4\pi}}\,(g_\pi L_\pi + g_\nu L_\nu), \qquad T(E2)_\mu^2 = e_\pi P_\mu^2(i)_\pi + e_\nu P_\mu^2(k)_\nu, \tag{2}$$

respectively, where

$$P_\mu^r(i) = \sqrt{5}\left[b_{ki}^\dagger \tilde{b}_{ki}\right]_{0\mu}^{0r}, \quad P_\mu^r(k) = \sqrt{15/2}\left[b_{ki}^\dagger \tilde{b}_{ki}\right]_{\mu 0}^{r0}, \quad r = 1, 2,$$
$$L_\pi = \sqrt{5}P_\sigma^1(i) \quad L_\nu = \sqrt{8/3}P_\sigma^1(k) \;. \tag{3}$$

In the above, e_π (e_ν) is the proton (neutron) effective charge and is fixed at 0.24 eb (0.20 eb) globally for all the rare-earth nuclei examined here. The g's are the g-factors in the S-D subspace; we take g_π=1.0 μ_N and g_ν=0 for all nuclei examined here.

In Fig. 1, we plot the experimental and calculated energies of the 2_1^+ and 1_1^+ states as a function of the factor $P \equiv N_p N_n/(N_p + N_n)$ where N_p (N_n) are the valence proton (neutron) numbers, respectively. The P scheme was introduced by Casten et. al [18, 1]) and can effectively display the global systematics. In a separate paper [17], we shall discuss the detailed level structures of the rare-earth nuclei. There we will show that

nuclei with the same P factor exhibit a correlation between the ground-band structure and that of mixed-symmetry states because they have the same $Q_\pi \cdot Q_\nu$ contribution, while prominent discrepancies in the β and γ excitation states may be largely attributed to the different pairing for protons and neutrons.

One can see the quality of the fit to the energies in Fig. 1. Notice that both the 2_1^+ and 1_1^+ states show orderly variation with a nearly constant energy gap between the two states (except for ^{154}Gd). These trends are also present in other states expected to be in the symmetric and mixed-symmetric classes [17].

The corresponding wavefunctions are used to compute the $E2$ and $M1$ transitions.The $B(E2)$ values and the summed $B(M1)$ strengths from this calculation are shown in Figs. 2a' and 2b', while the corresponding data are shown in Figs. 2a and 2b. The curves are the empirical relations presented in Ref. [1] that summarize the approximate behavior of the data. The $B(E2)$ and $B(M1)$ strengths are reproduced quantitatively by the calculations. Thus, we find theoretical evidence for the approximate universal behavior of $E2$ and $M1$ strengths exhibited by the data. Furthermore, we observe that even the deviations from universality exhibited by the data (for example, the $M1$ saturation is sharper and occurs at least 1 unit of P lower than that for the $E2$ strength (which never completely saturates)), is reproduced *quantitatively* by the calculations without parameter adjustment.

In Figs. 2c and Figs. 2c', we have plotted the ratio $E(4_1^+)/E(2_1^+)$ as a function of P. This quantity is also seen to exhibit an empirical variation with P that is similar to that of the $E2$ and $M1$ strengths, and it is also quantitatively reproduced by these calculations. Since we expect this ratio to be sensitive to the $Q_\pi \cdot Q_\nu$ interaction, this is an expected result given the success of the preceding calculations and our previous assertion that the $Q_\pi \cdot Q_\nu$ term is the most important factor governing the relationship between the properties of the symmetry and mixed-symmetry states.

Finally, we address the question of how important the variation of effective interaction parameters is to the success of these calculations. In Figs. 2a"–2c" we repeat the

calculations of Figs. 2a'–2c', but with a *fixed set of parameters for all nuclei*: $G'_{0\pi}$=-0.074 MeV, $G'_{0\nu}$=0.020 MeV, $B'_{2\pi}$=-0.001 MeV, $B'_{2\nu}$=0.047 MeV, and $B_{2\pi\nu}$=-0.243 MeV. Remarkably, these calculations are also in quantitative agreement with observations, differing only in minor details from the previous calculations in which the effective interactions have a weak A dependence. Thus, the quantitative reproduction of $E2$ and $M1$ strengths in the rare earth nuclei is an inherent feature of the FDSM: we reiterate that in Fig. 2, no parameters have been adjusted to either the $E2$ or $M1$ strengths. It should be noted that the small positive value of the renormalised $G'_{0\nu}$ means that the neutron quadrupole pairing is strong, thus implying that higher angular momentum pairs may also play some role.

Let us note that the successes of these calculations depend on the separation of particles into abnormal and normal-parity orbitals in the FDSM valence space, with only the particles in the normal-parity orbitals contributing directly to the $Sp(6) \times SO(8)$ collectivity. This separation has been specified by prior considerations wholly unrelated to the present discussion of $E2$ and $M1$ strengths. That the resulting theory describes quantitatively the global behavior of these collective strengths without parameter adjustment is evidence of the FDSM assumption that normal parity and abnormal parity orbitals play fundamentally different roles. Evidence for this separation has been presented before in the systematics of nuclear masses and deformation [19, 20, 10]. Such a division of the roles of normal and abnormal parity orbitals is not within the framework of the IBM, but its effect would be to lower the effective d boson number in the region of deformed nuclei. Perhaps adjusting the d-boson number may be an empirical way to obtain an IBM-2 description of these systematics.

In summary, the observed approximate universal behavior of nuclear collective symmetry and mixed-symmetry states is examined here. The F-spin formalism of the Interacting Boson Model suggests an elegant classification of these states, but an IBM-2 model with F-spin symmetry may not easily account for the universality exhibited by the corresponding $M1$ and $E2$ strengths. It is thus interesting to inquire whether by allowing

F-spin mixing the IBM-2 can address the correlation between the $M1$ and $E2$ strengths. What we have demonstrated here is that the FDSM can quantitatively reproduce the $M1$ and $E2$ strengths as well as the energy ratio $E(4_1^+)/E(2_1^+)$: the calculations can reproduce both the approximate universality and its subtle deviations without parameters adjusted to either the $M1$ or $E2$ strengths. This suggests that a properly chosen n–p quadrupole interaction in a symmetry-truncated fermion model can simultaneously account for the properties of symmetry and mixed-symmetry states. These results depend non-trivially on the separation of normal and abnormal parity orbitals, and represent another piece of evidence for this separation.

We believe the present conclusions to be rather general in nature. The FDSM illustrates the physics transparently and economically, but *any* fermion model leading to a good collective subspace and with realistic effective interactions suited to that subspace should be able to account for this behavior of the collective strengths. It will be important to see whether other fermion models can accommodate the approximate universality of symmetry and mixed-symmetry states in as simple a manner as presented here.

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Figure Caption

Fig. 1 Experimental and calculated energy levels for the first 2^+ (lower points) and 1^+ (upper points) states in selected even–even rare earth nuclei where orbital M1 strengths have been measured. The symbols denote isotopes with the same meanings as in Fig. 2.

Fig. 2 Comparison of experimental and theoretical strenghs for $B(E2)$ and summed $B(M1)$ for the rare-earths. Both are plotted as functions of P. The data which appear in the left column are taken from [1] ,[21] and [22]. The points with different symbols in (a')-(c') and (a")-(c") are theoretical results for different isotopes. For comparison, curves from the same empirical relation are used here, namely $B(E2, M1) = a_1 + a_2/[1 + exp((c - P)/d)]$ (In (a)–(a"), a_1=1.3, a_2=1.1, c=5.45 and d=0.57. In (b)–(b"), a_1=0.36, a_2=2.2., c=4.1 and d=0.32. In (c)–(c"), a_1=1.3, a_2=1.9., c=3.3 and d=0.49.) are also plotted. The symbols in (c)–(c") have the same meaning as in the $E2$ and $M1$ cases. The theoretical results in column 2 correspond to the parameters which can best fit the data. Finally, the theoretical results of column 3 correspond to constant values of the effective interaction parameters.